also suggests that considerable care must be taken in coupling the cavity to the measurement system. A tradeoff will have to be made to keep the coupling as loose as possible to preserve the high Q while retaining sufficient coupling to obtain a readable signal.

Microwave cavity techniques have been used widely in pulsed (decaying afterglow) plasma studies where the plasmas are usually quiet and well-behaved. If a plot of density versus time in the afterglow is to be obtained, then it is necessary that the plasma conditions be repeatable from pulse to pulse. In dc discharges many more problems arise. Due to the high electron temperatures, the collision frequencies in such discharges are usually large and can wash out the cavity resonances. Furthermore, the fluctuations, striations, etc., that are commonly found in dc discharges, as well as the microwave radiation directly into the cavity and the possible excitation and interaction of plasma modes, can contribute an excessive amount of noise to the detector signal and can make meaningful measurements impossible. Under such circumstances, it is sometimes possible to crowbar off the discharge for a short period of time, make measurements in the afterglow, and extrapolate back to the switch-off time to obtain estimates of the plasma density.

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Automatic Digital Method for Measuring the Permittivity of Thin Dielectric Films

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Abstract—One of the most promising techniques for measuring the electric permittivity at microwave frequencies of thin dielectric materials of the order of 0.1 to 10 μm, is the cavity perturbation method. For thin films of this type, it is necessary to determine accurately and display small changes in the resonant frequency and Q factor of the cavity in the presence of the material sample.

A circuit for the simultaneous measurement and digital readout of the resonant frequency and Q factor of microwave cavity is described. For the resonant frequency measurement, a very efficient automatic frequency circuit, with a homodyne modulation-detection bridge and frequency stabilization loop, is applied. Theoretical analysis and experimental results with this circuit show that an accuracy of 5×10^{-7} can be achieved in the resonant frequency measurement.

For measuring the Q factor, two similar circuits are described. The technique is based on measuring the phase shift of the envelope of an amplitude modulated microwave signal when this signal is transmitted through a resonant cavity at resonance. Although an accuracy of 0.5 percent in the Q factor can be achieved, it is shown that the main limiting factor in both circuits is the accuracy of phase shift determination at RF frequencies.

Manuscript received March 1, 1971; revised July 13, 1971.
The authors are with the Department of Electrical Engineering, University of Manitoba, Winnipeg, Canada. This work was supported in part by the National Research Council of Canada, under Grants A-3326 and A-7240, in part by the Defence Research Board of Canada, under Grants 6801-37 and 3801-42, and in part by the Faculty of Graduate Studies, University of Manitoba.

I. Introduction

THE permittivity of thin dielectric films has been of increasing interest due to its importance in the properties of semiconductor thin films and integrated circuits at microwave frequencies. More recently, the properties of thin biological films have received considerable attention since their behavior at microwave frequencies is related to microwave biological hazards [1]. Other applications are also encountered in the paper and textile industries where knowledge of the permittivity or moisture content is essential for quality control. Because of the high accuracies and large number of measurements usually required, a reliable and accurate method for measuring the permittivity of thin dielectric films or sheet materials is naturally of great interest, particularly if the method is digital and automatic.

The most suitable and accurate method developed so far for measuring dielectric properties is the cavity perturbation technique [2]. Other techniques based on the perturbation method have also been reported [1], [3], while other methods using a partly filled cavity [4] or a strip line structure [5] have been proposed. However, these methods are tedious and time consuming and measurement results are usually less accurate. In the millimeter wavelength region, Fabry–Perot resonators have been employed [6], but the measured film thicknesses were not less than 0.1 mm.

The method presented in this paper is valid for very thin films in the range of 0.1 to 10 μm as proved theoretically, and checked experimentally at X band for thicknesses of 1 to 5 μm. The method is automated and may be used for continuous and simultaneous measurement of the real (ϵ') and imaginary (ϵ'') parts of the complex permittivity. The method is based on a digital readout of the resonant frequency and Q factor of a test cavity with and without the dielectric film material. It is shown that ϵ' can be measured with reasonable accuracy (about 3 percent or better) depending on the film thickness and microwave frequency. Furthermore, since the Q factor of the cavity can be measured with accuracy as high as 0.5 percent, the method is also adequate for measuring larger values of ϵ'' with good accuracy.

II. Principle of Operation

The cavity perturbation method [2] is employed here to determine the permittivity ($\epsilon = \epsilon' - j\epsilon''$) of thin dielectric films. The main assumption of this method is that the dimensions of the sample are small compared to the wavelength which is always the case for thin films. If the sample surface lies across the cavity and everywhere tangential to the electric field, then ϵ' is given by [2]

$$\epsilon' = K \frac{L}{\tau} \frac{\Delta f}{f_{0s}} + 1 \tag{1}$$

where L is the cavity length, τ is the film thickness, Δf is the frequency shift $f_{0s} - f_0$, and f_{0s} and f_0 are the resonant frequencies of the cavity with and without the sample. K is a constant determined by the film geometry and cavity cross section and is equal to unity when the whole cross section is occupied by a film. Moreover, K is less than unity when the cross section is partially filled with a film and is larger than unity when a capacitive or resonant diaphragm is loaded with a dielectric film and inserted into a cavity [1]. Similarly,

$$\epsilon'' = K \frac{L}{2\tau} \left[\frac{1}{Q_{Ls}} - \frac{1}{Q_{L0}} \right] \tag{2}$$

where Q_{Ls} and Q_{L0} denote the loaded Q factor of the cavity with and without the sample, respectively. Before discussing practical circuits for measuring ϵ' and ϵ'', it is appropriate to present a brief analysis of the various factors that affect the measurement accuracy.

III. Discussion of Accuracy

The error in evaluating ϵ' may be calculated by the relation

$$d\epsilon' = K \frac{\Delta f}{f_{0s}} \frac{1}{\tau} dL + K \frac{L}{\tau} \left[\frac{\Delta f}{f_{0s}} \left(\frac{d\tau}{\tau} + \frac{df_{0s}}{f_{0s}} \right) + \frac{d\Delta f}{f_{0s}} \right]. \tag{3}$$

In most practical cases, the last factor is at least ten times larger than the sum of the other three factors. For very thin films, the error related to the accuracy of measuring the film thickness has a significant effect on the overall accuracy. However, in all cases the accuracy of measuring the cavity resonant frequency shift Δf is the main factor limiting the accuracy of determining ϵ'. In most practical circuits, this factor is limited by the stability of the microwave oscillator and the accuracy of measuring the resonant frequency of the test cavity [2]. This factor is considerably improved in the proposed method without improving the accuracy of the resonant frequency measurement (df_{0s}).

Similarly, the error in evaluating ϵ'' is given by the relation

$$d\epsilon'' = \frac{KL}{2\tau} \left[\frac{1}{Q_{Ls}} - \frac{1}{Q_{L0}} \right] \left[\frac{dL}{L} + \frac{d\tau}{\tau} \right] + \frac{KL}{\tau} \frac{dQ_L}{Q_L^2} \tag{4}$$

which indicates that the main source of error lies in the accuracy of measuring the Q factor. A comparison between existing methods for measuring Q [2] indicates that the phase comparison method is the most promising. Here a microwave signal is amplitude modulated and the envelope of phase modulation is compared at the input and output terminals of the cavity [7], [8]. It is expected that high accuracy can be achieved with a digital phase shift measurement method [9], [10].

IV. Digital System for Resonant Frequency-Shift Measurement

A block diagram of the proposed scheme is based on the cavity perturbation method and is shown in Fig. 1. Here a continuous signal from a microwave oscillator is fed into a microwave bridge of the type used for ampli-

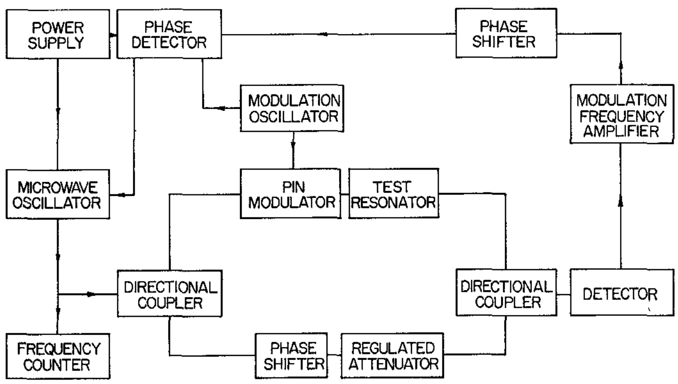

Fig. 1. Block diagram of the resonant frequency-shift measurement setup.

tude, phase or reflection coefficient measurement by the homodyne method [11]–[13]. One part of the signal goes through a phase shifter and a regulated attenuator to a detector. The other part is 10 dB smaller and is amplitude modulated by a p-i-n diode modulator and is then transmitted through the test cavity and is coupled to the other 10-dB directional coupler. As will be shown later, the signal after detection has a modulation frequency component whose amplitude and phase depend on the difference frequency between the microwave oscillator frequency and the resonant frequency of the cavity. This signal is amplified and then compared with a reference signal from a modulating oscillator in a phase detector. The output dc signal from this phase detector, which is a function of the above mentioned difference frequency, is finally applied in series with the supply voltage of a frequency tuning electrode in order to control the frequency of the microwave oscillator. The new resonant frequency is displayed on a frequency counter and utilized in calculating ϵ'.

In order to analyze the system we note that the detector input is the sum of the signals from the phase shifter and cavity channels, i.e.,

$$E = E_c e^{j\omega t + \phi_c} + E_{sc} \left| T(\omega) \right| (1 + m \cos \omega_m t) e^{j[\omega t + \phi_{sc} + \psi(\omega)]} \quad (5)$$

where

- E_c carrier signal fed by phase shifter,
- E_{sc} subcarrier signal equal to the amplitude of the signal at the input of the cavity,
- m modulation index,
- ω microwave frequency in radians,
- ω_m modulation frequency in radians,
- $T(\omega)$ $= |T(\omega)| \underline{/\psi(\omega)}$ = complex transmission coefficient of cavity,
- ϕ_c, ϕ_{sc} phase of carrier and subcarrier signals, respectively.

Since the level of carrier signal is high (microwave power higher than 1 mW), the detection may be considered linear [12]. The output of the linear detector is hence

TABLE I

q	$G_{q+1,1}$
1	m
2	$2m$
3	$3m\left(1 + \frac{m^2}{4}\right)$

$$\overline{E} = [E_c^2 + E_{sc}^2 \left| T^2(\omega) \right| (1 + m \cos \omega_m t)^2 - 2E_c E_{sc} \left| T(\omega) \right| (1 + m \cos \omega_m t) \cos (\theta + \psi)]^{1/2} \quad (6)$$

where $\theta = \pi + \phi$ and ϕ is the phase angle between carrier and subcarrier at input plane of cavity. The output signal at the modulation frequency is hence given by [12]

$$E(\omega_m) = - E_{sc} \left| T(\omega) \right| \left[m \cos (\theta + \psi) + \sin (\theta + \psi) \cdot \sum_{q=1}^{\infty} \left(\frac{E_{sc}}{E_c} \right)^q \frac{P_q'}{q(q+1)} G_{q+1,1} \right] e^{j(\omega_m t + \phi_m)} \quad (7)$$

where

- P_q' $= dP_q[\cos (\theta + \psi)]/d[\theta + \psi]$,
- P_q legendre polynomials of order q and $G_{q+1,1}$ are special functions tabulated in [12]. For the sake of convenience, we list three terms of this function in Table I,
- ϕ_m relative phase of modulation frequency signal.

Equation (7) indicates that the detector output at the modulating frequency is a function of the modulus of the cavity transmission coefficient and the phase angle ϕ between the two channels. Hence it is a function of the phase shifter position and argument of the cavity transmission coefficient. It should be noted that the transmission coefficient for a cavity near resonance is given by

$$T(\omega) \simeq \frac{T(\omega_0)}{1 + j2Q_L\left(\frac{\omega - \omega_0}{\omega}\right)}, \qquad \omega \simeq \omega_0 \quad (8a)$$

hence

$$\left| T(\omega) \right| = \frac{T(\omega_0)}{\left[1 + 4Q_L^2\left(\frac{\omega - \omega_0}{\omega}\right)^2\right]^{1/2}} \quad (8b)$$

$$\psi(\omega) = - \arctan 2Q_L\left(\frac{\omega - \omega_0}{\omega}\right) \quad (8c)$$

where

- $T(\omega_0)$ cavity transmission coefficient at resonance (real),
- ω_0 resonant frequency of the cavity,
- Q_L loaded cavity Q factor.

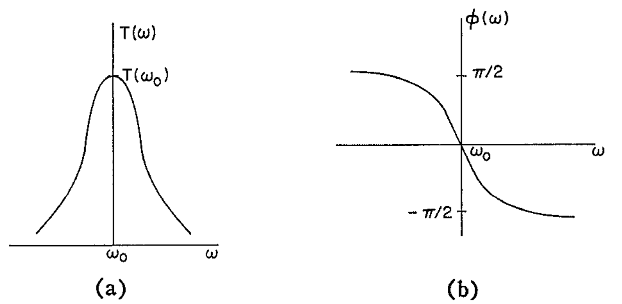

Fig. 2. Modulus and argument of the cavity transmission coefficient.

Equations (8b) and (8c) are plotted in Fig. 2. As a result the behavior of $E(\omega_m)$ near ω_0 is given by

$$E(\omega_m) = -E_{sc}\frac{T(\omega_0)}{\left[1+4Q_L^2\left(\frac{\omega-\omega_0}{\omega}\right)^2\right]^{1/2}}$$

$$\cdot\left\{m\cos\left[\theta-\arctan 2Q_L\left(\frac{\omega-\omega_0}{\omega}\right)\right]\right.$$

$$+\sin\left[\theta-\arctan 2Q_L\left(\frac{\omega-\omega_0}{\omega}\right)\right]$$

$$\left.\cdot\sum_{q=1}^{\infty}\left(\frac{E_{sc}}{E_c}\right)\frac{P_q'}{(q+1)q}G_{q+1,1}\right\}. \tag{9}$$

As shown elsewhere [11], [12], the ratio E_{sc}/E_c must be less than 0.1 to have homodyne circuit advantages. Hence all the terms in the sum, except the first, may be neglected. Equation (9), with θ replaced by $\pi+\phi$, then reduces to

$$E(\omega_m) = E_{sc}m\,|T(\omega)|\left\{\cos[\phi+\psi(\omega)] - 0.5\frac{E_{sc}}{E_c}\sin^2[\phi+\psi(\omega)]\right\} \tag{10}$$

and at resonance, where $|T(\omega)| = T(\omega_0)$ and $\psi(\omega)=0$, we have

$$E(\omega_m)\,|_{\text{res}} = E_{sc}mT(\omega_0)\{\cos\phi - 0.5(E_{sc}/E_c)\sin^2\phi\}. \tag{11}$$

In order for $E(\omega_m)$ to vanish at resonance, ϕ must satisfy

$$\frac{\cos\phi}{\sin^2\phi} = 0.5E_{sc}/E_c \tag{12}$$

whose solutions are denoted by ϕ_0 and given in Table II for different values of E_{sc}/E_c.

Table II shows that for the ratios $(E_{sc}/E_c)\leq 30$ dB even quite a large change in E_{sc}/E_c requires only a small adjustment of the relative phase shift ϕ_0 to satisfy (12). This is a substantial advantage of the method, described in detail in Section VI.

The modulation frequency signal versus microwave frequency, corresponding to values of ϕ_0 given in Table II, is shown in Fig. 3. For these values, the equation describing the curve simplifies to

$$E(\omega_m)\,|_{\phi=\phi_0} = \pm E_{sc}m\frac{T(\omega_0)}{\left[1+4Q_L^2\left(\frac{\omega-\omega_0}{\omega}\right)^2\right]^{1/2}}\cdot\sin\arctan 2Q_L\left(\frac{\omega-\omega_0}{\omega}\right). \tag{13}$$

The curves in Fig. 3 were calculated for typical values of $T(\omega_0)$, Q_L, E_{sc}/E_c and a mixer-detector circuit. This signal may be used to lock the microwave oscillator frequency to the resonant frequency of the cavity. The signal slope near resonance is proportional to the cavity

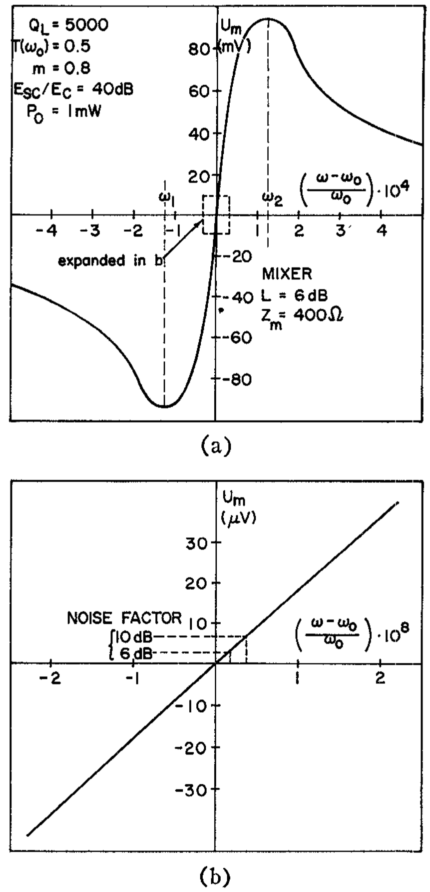

Fig. 3. Modulation frequency signal amplitude versus microwave frequency.

TABLE II

E_{sc}/E_c	20 log (E_{sc}/E_c) (dB)	ϕ_0 (degrees)
0.1	20	±87.14
0.05623	25	88.39
0.0316	30	89.09
0.01	40	89.71
0.00316	50	89.81

phase characteristic while the locking bandwidth $\omega_2-\omega_1$ can be found from the condition

$$\frac{dE(\omega_m)\,|_{\phi=\phi_0}}{d\omega}=0$$

which leads to the solution

$$\omega_{1,2}=\omega_0\pm\frac{\pi}{4Q_L}\,\omega_0. \tag{14}$$

V. Practical Procedure

Before any film measurements are carried out, it is necessary to adjust the circuit in the following sequence.

1) CW oscillator is turned on with frequency stabilization loop opened.

2) Phase shifter channel turned off by inserting about 50-dB attenuation (setting 50 dB at the regulated attenuator).

3) The oscillator is tuned to the resonant frequency of the cavity. This is achieved by observing with an RF millivoltmeter the output signal at the modulation frequency after the amplifier.

4) Attenuation in phase shifter channel decreased to zero.

5) Phase shifter position adjusted to obtain minimum output signal at the modulation frequency. During this procedure, the frequency of the oscillator should be kept as close to the cavity resonant frequency as possible.

6) Frequency stabilization loop closed.

The actual measurement requires a readout of the frequency counter (see Fig. 1) before and after inserting the dielectric film into the cavity. It should be noted that no additional adjustments are necessary when the film is very thin and the resulting frequency shift is less than the locking bandwidth $\omega_2-\omega_1$ given in (14). However, when the film is too thick or has a large ϵ', the stabilization circuit may be put to work after the dc voltage control electrode is manually adjusted or a special search oscillator is applied. This would not be necessary if the dielectric film is thin and the frequency shift is small, for which the method is most suited.

VI. Main Sources of Error

The main sources of measurement error can be divided into three groups: errors related to the frequency stabilization circuit; variations of the cavity resonant frequency during measurement; and the frequency counter.

A. Frequency Stabilization Circuit

Following the procedure described in Section V, the accuracy of the resonant frequency-shift measurement is not limited by the accuracy of the cavity resonant frequency measurement. This is because after the initial adjustments are made, the control circuit keeps the oscillator frequency equal to the cavity resonant frequency plus (or minus) a certain value resulting from the limited possibilities of finding the exact resonant frequency of the unloaded cavity. This value remains exactly the same when the cavity is loaded due to the film sample and does not, therefore, affect the difference between the two measured oscillator frequencies. This is true provided that all other parameters of the control circuit do not change at the same time. From the previous analysis it can be seen that the parameters which can influence the accuracy of the frequency shift measurement are: variations in the phase shift between channels (ϕ); variations in the phase shift required for (13) to be valid (i.e., phase shift required in order to have zero output at resonance); noise level; variations in the control loop gain; and fluctuations in the oscillator power level.

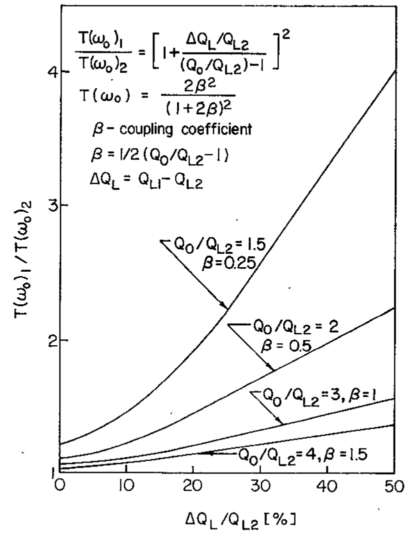

Fig. 4. Changes of the transmission coefficient of the cavity at resonance versus loaded Q factor variations.

Variations in ϕ can occur if either carrier or subcarrier channel lengths change or if the frequency changes when the two channels differ in electrical length. The length can change with vibrations or temperature fluctuations, but in practice this effect can be easily minimized under modern laboratory conditions. Also the changes of length due to frequency fluctuations are usually minimized by making both channels as equal in electrical length as possible, which over a limited frequency range is not a serious problem.

Variations in the phase shift required for zero output at resonance are due to changes in the transmission coefficient $T(\omega_0)$ at resonance. They are caused by the fact that Q_L changes in the presence of the measured film. Fig. 4 shows a theoretical plot of the variations in $T(\omega_0)$ versus variations in Q_L with the cavity coupling coefficient β as a running parameter. From Table II, or more accurately from (8), the changes in a required phase shift angle ϕ_0 caused by changes in $T(\omega_0)_1/T(\omega_0)_2$ may be found. From known $\Delta\phi$, the corresponding frequency change can be found after some manipulations of (9) which simplifies to

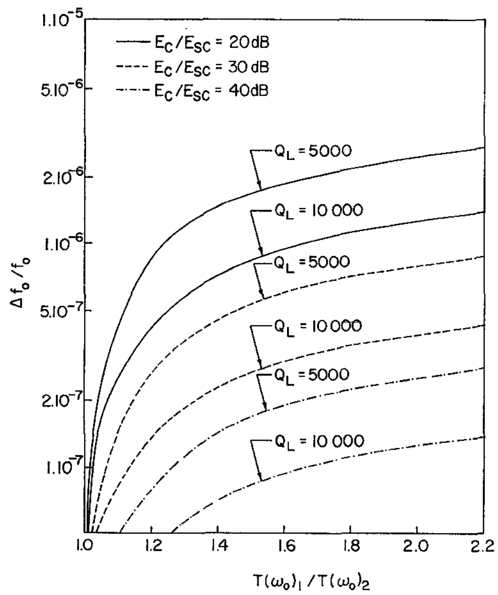

Fig. 5. Relative resonant frequency measurement error versus variation of the transmission coefficient of the cavity at resonance.

$$\frac{\Delta\omega}{\omega_0} \simeq \frac{\tan \Delta\phi}{2Q_L}. \tag{15}$$

Fig. 5 shows the measured changes in the resonant frequency caused by a change in the transmission coefficient at resonance and with the ratio of carrier to subcarrier signal as a running parameter. Thus the error related to a "zero phase shift" change can be evaluated from Figs. 4 and 5 where E_{sc}/E_c can be chosen as 40 dB for most practical cases. The resulting changes in $T(\omega_0)_1/T(\omega_0)_2$ are seldom larger than 1.5 as can be seen from Fig. 4, so that the error in $\Delta f_0/f_0$ does not exceed 2×10^{-7} for $E_{sc}/E_c=40$ dB and 5×10^{-7} for $E_{sc}/E_c=30$ dB.

The noise level may be kept reasonably low by employing a high-modulation frequency and a low-noise diode for homodyne detection. For carrier to subcarrier level as low as 40 dB, a 1.0° phase change in any channel causes a 3-dB output signal at the detector [11]. This results in the oscillator frequency being kept close to the cavity resonant frequency as evident from

$$\arctan 2Q_L \frac{\omega - \omega_0}{\omega_0} = 0.1°$$

hence

$$2Q_L \frac{\omega - \omega_0}{\omega_0} = 0.002$$

and

$$\left(\frac{\Delta f_0}{f_0}\right)_{\text{noise}} = \frac{1}{Q_L} 10^{-3}$$

which leads to the conclusion that the frequency instability due to noise factor is in the order of 5×10^{-7} to 1×10^{-7} for Q_L in the range 5×10^3 to 10^4.

The variations in the control loop gain and oscillator power level may be practically eliminated by the feedback loop. The combined effect of all parameters on the variation in the frequency shift $\Delta(\Delta f)/f_0$ lies in the range 10^{-6} to 5×10^{-7} for most practical cases.

B. Cavity Resonant Frequency

Variations in the cavity resonant frequency during measurement are usually caused by room temperature fluctuations and deviations in reassembly after insertion of film samples. These two factors are important and special care to minimize their effects should be taken.

C. Frequency Counter

The errors caused by the frequency counter can be serious unless its accuracy is 5×10^{-8} or better.

VII. Digital System for Bandwidth Measurement

Since ϵ'' is related to the Q factor of the cavity, and hence the bandwidth, it is convenient to employ a method that allows the simultaneous measurement of resonant frequency and Q factor. The phase comparison method [2] has been selected for this since it also permits high-accuracy automated measurements with a digital readout and does not require frequency sweep. The main limitation on the accuracy of this method lies in the phase shift measurement where modern digital techniques may be employed to measure phase shift as small as 0.1° [9], [10], [14], [15].

The principle of the method is based on measuring the phase shift of an envelope of an amplitude modulated signal as it does through a resonant cavity. Assuming that the amplitude of this signal at the input terminals of the cavity is given by

$$E_{\text{in}} = E_0(1 + m \cos \omega_m t) \cos \omega t \tag{16}$$

the amplitude at the cavity output is

$$E_{\text{out}} = E_0 \frac{m T_0(\omega)}{\left[1 + 4Q_L^2 \left(\frac{\omega_m}{\omega_0}\right)^2\right]^{1/2}} \cdot \cos\left[\omega_m t - \arctan\left(2Q_L \frac{\omega_m}{\omega_0}\right)\right] \tag{17}$$

where the cavity transmission coefficient at resonance in (8) has been used and m is the modulation index.

Figs. 6 and 7 show two schemes for automatic measurement of the Q factor. Fig. 6 corresponds to a constant modulating frequency circuit where the phase shift between the input and output modulation signals is given by

$$\Delta\psi = \arctan\left(2Q_L \frac{\omega_m}{\omega_0}\right) \tag{18}$$

where Q_L may be determined from (18) once ω_m and ω_0 are known and $\Delta\psi$ measured accurately using a counter

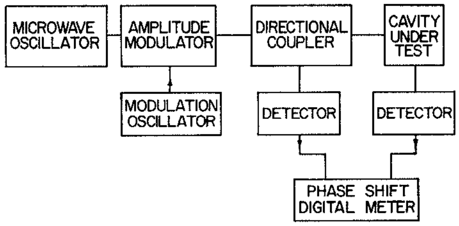

Fig. 6. Block diagram of the Q factor measurement setup (constant modulation frequency circuit).

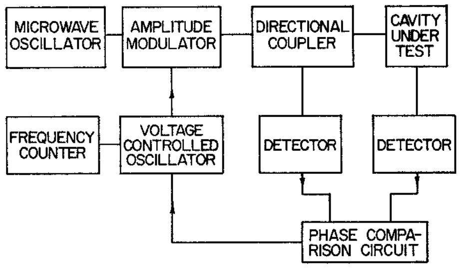

Fig. 7. Block diagram of the Q factor measurement setup (constant phase shift circuit).

[8]–[10]. It should be noted that the stability and accuracy of measuring ω_0 is improved by applying the frequency stabilization circuit used in conjunction with the frequency shift measurement. On the other hand, the amplitude modulation linearity is limited by the harmonics that influence the signal shape and introduce phase shift measurement errors.

Using (18), the overall accuracy of the Q_L measurement may be determined from the relation

$$\Delta Q_L = \frac{\tan \Delta\psi}{2\omega_m}\left[\Delta\omega_0 + \omega_0 \frac{\Delta\omega_m}{\omega_m}\right] + \frac{\omega_0}{2\omega_m}\left[(1+\tan^2\psi)\Delta(\Delta\psi)\right]. \quad (19)$$

For a circuit with frequency stabilization, the first term may be omitted. If $\Delta\omega_m/\omega_m = 10^{-4}$ and $Q_L \simeq 5\times 10^3$ at X band, $\omega_m \simeq 1$ MHz and $\Delta Q_L/Q_L$ is less than 0.5 percent for $\Delta(\Delta\psi) = 0.1°$ and 3.5 percent for $\Delta(\Delta\psi) = 1°$.

The second scheme corresponds to a 45° phase shift circuit as shown in Fig. 7. Here the sources of error and resulting accuracy are practically the same as in Fig. 6 but the calculation of the Q factor is simplified.

VIII. Experimental Results

The complete experimental setup is shown in Fig. 8 where the cylindrical TE_{012} mode test cavity (of 1.968-in length and 2.02-in diameter) is resonant at approximately 9.3 GHz. The measured Q_L for this cavity was found to be approximately 6×10^3.

The frequency stability of the circuit was 1×10^{-6}. This value could have been larger if the modulation frequency were lower because of the low-amplitude modulation efficiency of the p-i-n diode modulator used. As a result, the total error in the frequency shift measurement of the cavity resonant frequency was estimated to be no greater than 1.1×10^{-6}.

The bandwidth of the phase lock control loop was found to be about 2 MHz which is in good agreement with 2.3 MHz value obtained from (14).

The thickest Teflon film ($\epsilon' = 2.08$) which could be measured was 5 μm resulting in a 1-MHz frequency shift and a measurement accuracy of 1 percent based on (3). The accuracy for a 1-μm thick Teflon film was found to be better than 5 percent, although higher accuracy could still be achieved by improving the stabilization circuit.

The error in the Q factor measurement was estimated to be about 1.8 percent on basis of a 0.5° phase shift measurement accuracy. However, the loss tangent for low-loss materials (e.g., Teflon, polystyrene) was impossible to measure with satisfactory accuracy. For wet paper with $\epsilon'' = 0.1$, the measurement error was about 20 percent.

IX. Conclusions

An automatic modern digital technique for the simultaneous and contactless measurement of ϵ' and ϵ'' of thin dielectric films using a TE_{01n} cylindrical cavity has been presented. The range and expected accuracy of the measurement are very accurately predicted by the analysis described.

The ϵ' measurement is limited on the one hand by the accuracy of the frequency shift measurement, which determines the thinnest film than can be measured with reasonable accuracy, and on the other hand, by the locking bandwidth which determines the thickest film that can be measured. As an example for dielectric films with ϵ' in the 2.0–3.0 range, the thinnest and thickest films that can be measured with the cavity in the TE_{011} mode ($Q_L \simeq 5\times 10^3$) are given below.

Frequency Band (cm)	Maximum Film Thickness (μm)	Minimum Film Thickness (μm) 3-Percent Accuracy	10-Percent Accuracy
10	10	1.5	0.5
3	3	0.5	0.15
1.25	1	0.2	0.07

The smallest values of ϵ'' which can be measured to 10 percent accuracy depend on Q_L and the phase shift measurement accuracy as illustrated below.

Q_L	Phase Shift Measurement Accuracy	Minimum ϵ'' for 1-μm Thick Film with 10-Percent Accuracy
5000	0.1°	0.15
5000	0.5°	0.60
10 000	0.1°	0.075
10 000	0.5°	0.30

By introducing a search oscillator into the frequency-shift measurement, it may be possible to increase the

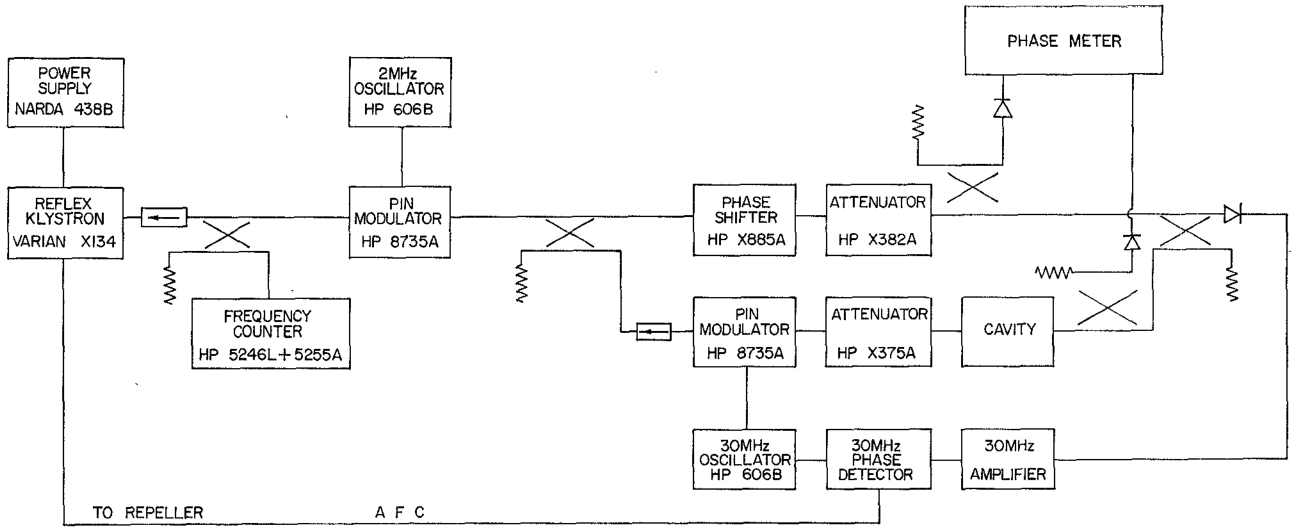

Fig. 8. Schematic diagram of the experimental setup.

measurement range to thicker films. This, as well as the possibility of continuous monitoring of dielectric sheet materials during technological processes (e.g., paper, textiles), are presently under investigation.

Acknowledgment

The authors wish to thank Prof. S. S. Stuchly, Prof. W. M. Boerner, Prof. G. O. Martens, and Prof. E. Bridges for many profitable discussions throughout this project.

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